Homework Set #7

Axiomatische Verzamelingentheorie
2013/14: 2nd Semester; block b
Universiteit van Amsterdam

`http://hugonobrega.github.io/teaching/AxVT/`

The *preferred method* for submitting homework solutions is by handing them in before the start of the *werkcollege* on Wednesday morning. Electronic submissions are also possible, by email to `ilin.juli (at) gmail.com` or `hugonobrega (at) gmail.com` before the deadline. These must be in a single, legible PDF file. It is also possible to hand in your homework by putting it into Julia's or Hugo's mailboxes at the ILLC at Science Park 107, but in this case the deadline for submission is at 10:45am on the day of the submission deadline (before Julia and Hugo leave for the *werkcollege*). It is important to respect the strict deadlines stated above; late homework will not be accepted.

This homework set is due on **Wednesday 21 May 2014, before the morning *werkcollege*.**

The homework will be graded by Friday 23 May 2014 and can be collected during an office hour (from 11-13; location to be announced). Please also write your e-mail address (legibly!) on your homework solution so that we can contact you by e-mail.

1. By transfinite recursion on the ordinals, we define *ordinal multiplication* as follows:

$$\begin{aligned}\alpha \cdot 0 &= 0,\\ \alpha \cdot (\beta + 1) &:= \alpha \cdot \beta + \alpha,\\ \alpha \cdot \lambda &:= \bigcup_{\xi \in \lambda} \alpha \cdot \xi \text{ (if } \lambda \text{ is a limit ordinal).}\end{aligned}$$

   Let α and β be ordinals of which at least one is a limit ordinal (note that this implies that $\alpha \cdot \beta$ is a limit ordinal). Show that $\mathrm{cf}(\alpha \cdot \beta)$ is either equal to $\mathrm{cf}(\alpha)$ or equal to $\mathrm{cf}(\beta)$.

2. Let κ, λ, and μ be cardinals. Show that

$$(\kappa^\lambda)^\mu = \kappa^{\lambda \cdot \mu}$$

   where the $\cdot$ on the right hand side of the equation denotes *cardinal multiplication* (not to be confused with ordinal multiplication from exercise 1.).

3. Assume that $2^{\aleph_0} = 2^{\aleph_1} = 2^{\aleph_2} = 2^{\aleph_3} = 2^{\aleph_4} = \aleph_5$ and that ${\aleph_\omega}^{\aleph_0} = \aleph_{\omega+3}$. Calculate the values of the following cardinal exponents: ${\aleph_8}^{\aleph_0}$, ${\aleph_2}^{\aleph_3}$, ${\aleph_9}^{\aleph_4}$, ${\aleph_{\omega+2}}^{\aleph_0}$, and ${\aleph_{\omega+7}}^{\aleph_0}$.